

MU120 RBB



The Open
University

Mathematics
and Computing
A first level
multidisciplinary
course

Open **Mathematics**



RESOURCE BOOK B

Units 6 – 9





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Prepared by the course team

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Unit 6 Maps

Question 1

Unit 6, Sec. 2.1 and 2.2

Use your Ordnance Survey (OS) map extract of the Peak District to do the following.

- (a) Identify the feature located at each of the following grid references:

168829 147844 136880 150829

- (b) Find the grid references of the following locations:

top of Crook Hill, north of Ladybower reservoir;

Druid's Stone, about 3 km north of Hollins Cross;

Blue John Cavern, about 2 km west of Castleton;

railway station about 1 km east of Hope.

Question 2

Unit 6, Sec. 2.1 and 2.2

The Ordnance Survey's national grid divides Great Britain into squares 100 km by 100 km. What size would a map of one of these squares be if it were drawn to a scale of 1 : 50 000?

Question 3

Unit 6, Sec. 2.2, 4.2 and 5

- (a) On your OS map extract, measure the straight-line distances from the cairn at grid reference 147874 to the following features, and then complete the table. Give the map distances to the nearest millimetre and the ground distances to the nearest 100 metres.

Feature	Grid reference	Map distance/cm	Ground distance/km
Guide post	160873		
Cairn	149882		
Madwoman's Stones	136880		
Shooting cabin	142887		

- (b) Instead of measuring the straight-line distances on the map, it is possible to calculate the ground distances from the grid references by using Pythagoras' theorem (see the diagram in Section 6.2 of the *Calculator Book*). Check your answers to part (a) using this method.

Question 4

Unit 6, Sec. 3.2

In 1994, magnetic north was 5° west of grid north on your OS map extract, and estimated to be decreasing by about $\frac{1}{2}^\circ$ every 3 years. Use this information to predict the direction of magnetic north relative to grid north on your OS map extract in 2004.

Question 5**Unit 6, Sec. 3.2**

To check her position, a walker takes two compass bearings. The first on the chimney at grid reference 164824 (on your OS map extract) comes out at 119° . The second on Win Hill (186850) is 83° . Where is she (assuming it is the year 1994 when, for this map, magnetic north was 5° west of grid north)?

Question 6**Unit 6, Sec. 3.3 and Calc. Book, Sec. 6.2**

A walk is planned to start and finish in the car park (122832) below Mam Tor. The route climbs to Mam Tor and then follows the path to Hollins Cross (136845). From there, the path turns north and descends to Backtor Bridge to join the road at 136855. It then follows the road to just past the T-junction where it turns left (123852) to follow the path up Harden Clough. The path meets the road from Barber Booth at 123837, and then follows the road back to the car park.

Use your OS map extract and Naismith's rule to estimate how long the walk will take.

Question 7**Unit 6, Sec. 4.1**

Using your OS map extract, estimate the change in height relative to the cairn, at grid reference 147874, for each feature listed below. Use your answers to Question 3 to estimate the distance from the cairn to each feature, and the average gradient of the straight line from the cairn to the feature.

Feature	Grid reference	Height/m	Change in height/m	Distance/m	Average gradient
Guide post	160873				
Cairn	149882				
Madwoman's Stones	136880				
Shooting cabin	142887				

Question 8**Unit 6, Sec. 4.4**

An old UK road atlas is drawn to a scale of 3 miles to 1 inch. What is the map scale expressed as a ratio? What area in hectares does 1 square inch represent?

In the imperial system of units, there are 12 inches to 1 foot, 3 feet to 1 yard and 1760 yards to 1 mile. 1 mile is equivalent to approximately 1.61 km.

Question 9**Unit 6, Sec. 4.4**

- An area of 50 hectares is represented on a map by an area of 1 cm^2 . What is the scale of the map?
- If the map scale were 1 : 100 000, what area in hectares would 1 cm^2 represent?

Question 10*Calc. Book, Sec. 6.1*

Use your calculator to plot a line graph representing the profile of the straight-line route between the cairns at grid references 147874 and 149882. Experiment to see the effect of different window settings on the shape of your graph. At what distance from the cairn at 147874 does the gradient become zero?

Question 11*Calc. Book, Sec. 6.1 and 6.2*

A walker plans the following walk:

Location	Grid reference
Yorkshire Bridge	197849
West bank of Ladybower Reservoir	197856
Near New Barn	190860
Saddle point on Win Hill	183850
Win Hill Pike	187851
Yorkshire Bridge	197849

- Enter appropriate eastings and northings on your calculator and draw a line graph representing the route of the walk.
- Carry out appropriate list arithmetic using Pythagoras' theorem to estimate the distance of each section of the walk, and then estimate the total distance of the walk.

Unit 7 Graphs

Question 1

Unit 7, Sec. 1.2

At 1999 exchange rates, £300 sterling was equivalent to 450 euros and 1000 euros was equivalent to £667 sterling.

- (a) Using these data, draw a graph to convert between pounds sterling (on the horizontal axis) and euros (on the vertical axis).
- (b) From your graph, convert £400 into euros, and 750 euros into pounds.
- (c) What type of relationship does your graph show?

Question 2

Unit 7, Sec. 1.2

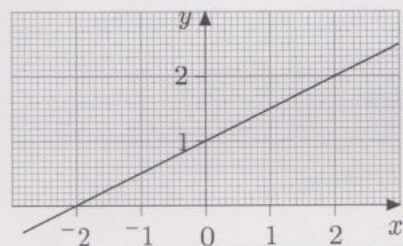
Two bird sanctuaries compare estimates of their bird populations to see which has the larger number of wild fowl. The bird population of sanctuary A is about 5200 at present and is increasing steadily at an average rate of 500 birds per year. That of sanctuary B is currently 6000 and is increasing steadily at an average rate of 300 birds per year.

- (a) Write down the word formulas that can be used to predict the bird populations in future years.
- (b) Draw the graphs showing the growth of the two bird populations over a six-year period from the present. When will the populations of the two sanctuaries be equal?

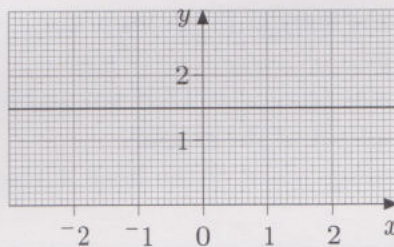
Question 3

Unit 7, Sec. 1.2, 1.3

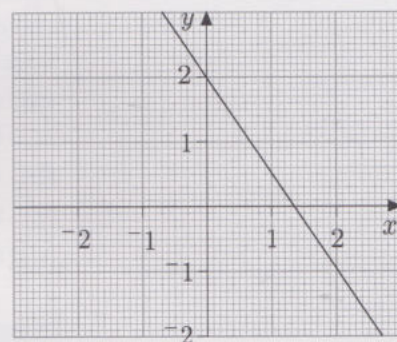
Find the gradient and the intercept of each of the following straight-line graphs.



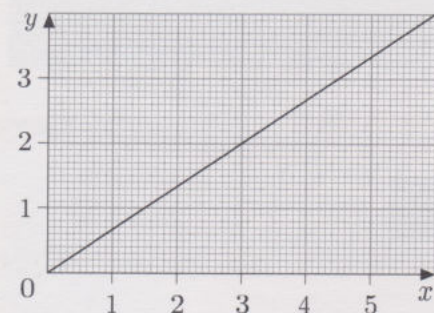
(a)



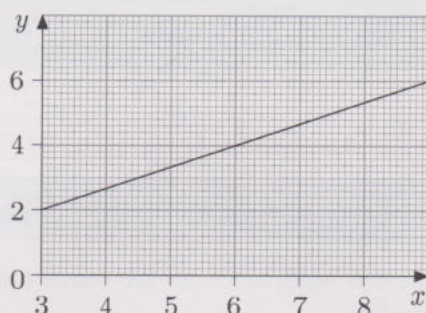
(b)



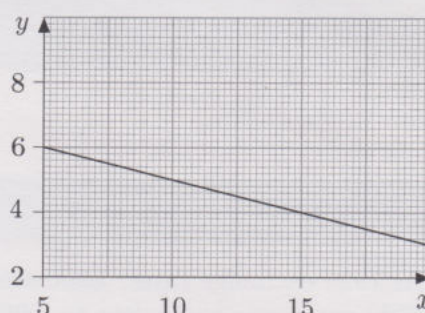
(c)



(d)



(e)



(f)

Question 4

Unit 7, Sec. 1.3

Complete the table below for the relationship where the y -coordinate is equal to the square of the x -coordinate ($y = x^2$).

x -coordinate	y -coordinate	Coordinate pair
-2.5	6.25	$(-2.5, 6.25)$
-1.5	2.25	$(-1.5, 2.25)$
-0.5		
0		
1		
2		
3		

Using your completed table, plot a graph of the relationship on squared paper. How does the gradient of the graph change as the value of the x -coordinate increases from -2.5 to 3?

Question 5

Unit 7, Sec. 2.1

A cross-channel ferry usually takes 2 hours to make the 40 km crossing from England to France. Assuming that the ferry leaves at 12 noon (all the times used are in GMT) and the speed of the ferry is constant during the crossing, draw a distance-time graph to represent the journey. From your graph find:

- when the ferry is 15 km from England, and when it is 35 km from England;
- where the ferry is at 12.15 pm and at 1.10 pm;
- the gradient of the graph;
- the average speed of the ferry during the crossing.

Question 6

Unit 7, Sec. 2.1

A ship travelling at about 10 knots (nautical miles per hour) directly towards a port radioed for a tug when it was 10 nautical miles away from the port. The tug set off from the port 30 minutes later, travelling towards the ship at about 5 knots. Draw the distance-time graphs for the ship and the tug on the same axes and hence predict where and when the vessels will meet. When should the tug start looking for the ship?

Question 7*Calc. Book, Sec. 7.1*

- (a) Using the data given in the answer (at the back of this Resource Book) to Question 6 for *Unit 6*, use your calculator to draw a distance–time graph for the planned walk around Mam Tor.
- (b) From the graph, estimate for which legs of the walk the fastest and slowest speeds are predicted.
- (c) Calculate the average speeds for each section of the walk.
- (d) What is the average speed for the entire walk?

Question 8*Calc. Book, Sec. 7.2*

Some recipe books give capacities in fluid ounces (fl. oz) while others use millilitres (ml). A dictionary says that 20 fl. oz (or 1 pint) is equivalent to 0.568 litres.

- (a) Calculate the number of millilitres equivalent to 1 fl. oz.
- (b) Use your calculator to display a graph that converts fluid ounces into millilitres. Choose window settings so that capacities up to 1 pint can be represented.
- (c) Use TABLE to estimate:
 - (i) the equivalent of 8 fl. oz in millilitres;
 - (ii) the equivalent of 200 ml in fl. oz.
- (d) Repeat these estimates using the TRACE facility.

Question 9*Unit 7, Sec. 2 and Calc. Book, Sec. 7.2*

You are planning a coach journey across France, and expect to average 95 km per hour.

- (a) Write down the formula relating the distance travelled in kilometres to the time travelled in hours, in a form suitable for your calculator.
- (b) Store the formula in your calculator, and use it to produce a table giving your expected distances every half-hour for $4\frac{1}{2}$ hours.
- (c) Suppose that you want to have a short stop after about two hours. How far would you have gone after travelling for two hours?
- (d) Suppose that there is not a convenient service area after two hours, but there is one at 175 km and another at 220 km. Use the stored formula on your calculator to estimate for how long you would have been travelling when you pass each of these service areas.

Question 10*Calc. Book, Sec. 7.2*

If a mass of P lb is equivalent to K kg, then P and K are related as follows:

$$K = 0.45P.$$

This is the formula for converting mass in pounds to mass in kilograms.

Write down a similar formula for converting mass in pounds to mass in stones (a stone is 14 lb). Use your calculator to produce a table corresponding to these formulas, which will allow you to convert directly from stones and half stones to kilograms.

Unit 8 Symbols

Question 1

Prep. Res. Book A and Unit 8, Sec. 2

Simplify the following expressions, which involve positive and negative quantities, and brackets:

- | | | |
|---|---------------------------------------|---------------------------------|
| (a) $4 - 3 + 5$ | (b) $5 + 8 - 15$ | (c) $-7a + 14a - 3a$ |
| (d) $4 - (-3)$ | (e) $6 + (-8)$ | (f) $12 - (+4)$ |
| (g) $121p + 43p - (-46p)$ | (h) 8×-7 | (i) $3 \times (+4) \times (-9)$ |
| (j) $-4 \times -6 \times -8 \times -12$ | (k) $-2 \times 4 \times -6 \times -3$ | (l) $-4 \div +8$ |
| (m) $\frac{-16q}{-4}$ | (n) $\frac{28}{-7} + (-4 \times -3)$ | |

Question 2

Unit 8, Sec. 1.2

- (a) In a greengrocer's shop, bananas are priced at 95p per kg. Find a formula which can be used to calculate the cost of any number of kilograms of bananas.
- (b) Customers are also charged 7p for a carrier bag. Amend your formula to include the purchase of a carrier bag. Then convert this formula to give the cost in £s.
- (c) Using the formula obtained in part (b), find the cost, in £s, of 2.6 kg of bananas in a carrier bag.

Question 3

Unit 8, Sec. 1.2

The cost of local telephone calls is 2.4p a unit. There is a standing charge of £15.30 a quarter. For a household that makes only local calls, find a formula relating the quarterly bill, in £s, to the number of units used.

Question 4

Unit 8, Sec. 1.2

If $p = 3q + 2$, find the value of p when

- (a) $q = 2$ (b) $q = 1.74$ (c) $q = -1.5$.

Question 5

Unit 8, Sec. 1.2

The dimensions of Noah's ark are given in Genesis 6, 14–15 as follows: 'Make thee an ark of gopher wood ... the length of the ark shall be three hundred cubits, the breadth of it fifty cubits, and the height of it thirty cubits.'

Calculate the dimensions of the ark in metres. (The formula for converting the number of cubits, C , into the number of metres, L , is $L = 0.46C$.)

Question 6*Unit 8, Sec. 2*

What is the value of $2x^2 - 3$ when $x = -2$?

What is the value of $\frac{2}{3}(4 + 3q)$ when $q = \frac{4}{5}$?

Question 7*Unit 8, Sec. 2*

(a) Express the following without $\times$ signs:

$$5 \times N, \quad 5 \times 6 \times z, \quad 20y \times 2.$$

(b) Simplify the following as far as possible:

$$2y - 7y, \quad 2 \times 5x + 4 \times 3x, \quad 6p + 7p^2 - 4p + 2p^2.$$

Question 8*Unit 8, Sec. 2*

Simplify the following expressions as far as possible:

(a) $7y + 4x - 3y$

(b) $p + 2r - q + 4p + q$

(c) $17\clubsuit - 9\spadesuit + 23\heartsuit - 5\clubsuit$

(d) 4 times k minus 13 times k plus s

(e) $2(A)^2 + (3A)A$

(f) $3ab + 3ab^2 - 2ba + b(ab)$.

Question 9*Unit 8, Sec. 2*

Remove all the brackets from the following:

(a) $2(3x - 1)$ (b) $4(k + 5)$ (c) $p(2q + p)$ (d) $2m(a - b + c)$

(e) $2b(b + b^2 - b^3)$ (f) $-3(x + 2y - z)$ (g) $\frac{1}{3}(6r - 9r^2)$.

Question 10*Unit 8, Sec. 2*

Multiply out these brackets:

(a) $(2 + x)(5 - y)$ (b) $(p + q)(x + y)$

(c) $(3 - y)(4 + y)$ (d) $(5f - 3)(2f - 1)$.

Question 11*Unit 8, Sec. 2*

Which of the following expressions are equivalent to $16p^2qr^3$?

(a) $4 \times 4p^2qr^3$ (b) $16r^3qp^2$ (c) $16pq^2r^3$ (d) $16p^3q^2r$

(e) $2 \times 2 \times 4 \times q \times p \times p \times r \times r \times r$ (f) $32p^2q^2r^3/2q$.

Question 12*Calc. Book, Sec. 8.2*

Press the 'Y=' key on your calculator and input the function $y = 4x - 2.3$.

(a) Use the method outlined in Section 8.2 to obtain y values when x takes the values 3, 13.72 and -101.5 .

(b) Plot the corresponding graph, and use TRACE and ZOOM to check your answers to part (a).

Question 13

Unit 8, Sec. 4.1

Consider the formula $p = 4.2q - 1.7$, which is to be rearranged to make q the subject.

- Which two operations have been applied to q to get $4.2q - 1.7$?
- Suppose you want to rearrange the formula to give q in terms of p ; write down the two operations that you would need to perform on $4.2q - 1.7$ to get q . Be careful about the order of the operations.
- Carry out the operations specified in (b) on the whole formula, and so write the formula in the form $q = \dots$

Question 14

Unit 8, Sec. 4.1

Use the method of Question 13 to make x the subject of the following equations:

- $y = x/3 + 9$
- $y = 5 + 12x$
- $y = 12 - 2x$.

Question 15

Unit 8, Sec. 4.2

- What operations are applied to x , and in what order, to obtain $2x + 5$? By reversing the operations and their order, solve the equation $2x + 5 = 8$.
- Substitute your value of x into the original equation to see if that value 'satisfies' the equation. If it does, your answer is correct.

Question 16

Unit 8, Sec. 4.2

Solve the following equations. In each case, substitute your answer back into the equation as a check.

- $3x - 2 = 2.5$
- $\frac{2}{3}x + 3 = 2$
- $2x + 3 + 2x - 1 = 20$
- $5(x - 2) = 8$.

Question 17

Unit 8, Sec. 4.2

Hansel is six years older than Gretel. In two years time he will be three times as old as Gretel. How old is Gretel now?

Question 18

Calc. Book, Sec. 8.4 and 8.5

On your calculator, display the graph of $y = x^3 + 3x^2 - 9x + 3$. Use any of the methods described in the *Calculator Book* to find:

- the value of y when $x = 2.25$;
- the values of x when $y = 10$.

Question 19*Calc. Book, Sec. 8.4 and 8.5*

An amateur gardener bought, by post, 17 rooted cuttings of fuschias. Postage and packing cost £2.36, and the total bill was £11.71.

- Using f to represent the price of one fuschia in pounds, write down an equation to model the transaction.
- On your calculator, enter the functions $y = 17x + 2.36$ and $y = 11.71$. Choose suitable WINDOW settings and draw the graphs.
- Use TRACE and ZOOM to find the x value (correct to 2 decimal places) of the point where the graphs cross.
- Look at your equation from part (a) and the functions used in part (b), and try to spot a connection between them. Hence find the cost of a single fuschia cutting (without postage and packing).

Question 20*Calc. Book, Sec. 8.4*

A firework, resting on the ground, is propelled vertically into the air so that its height, h , in metres after t seconds is given by

$$h = 5t - t^2.$$

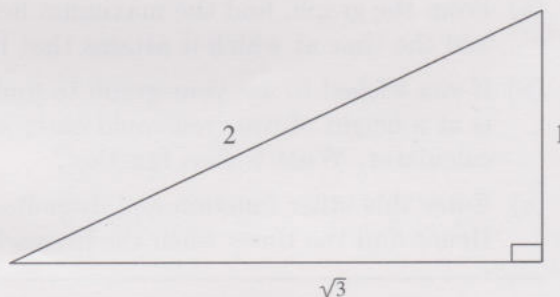
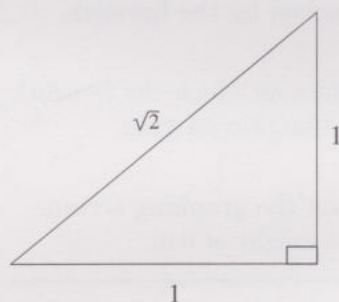
Display the graph of this function on your calculator. (Remember that you will have to change h to y , and t to x .)

- From the graph, find the maximum height reached by the firework, and the time at which it attains that height.
- If you wished to use your graph to find the times at which the firework is at a height of 6 m, you could enter another function on your calculator. What is that function?
- Enter this other function and draw its graph on the graphing screen. Hence find the times when the firework is at a height of 6 m.

Question 1

Calc. Book, Sec. 9.1

- (a) Use your calculator to evaluate the following expressions, giving the answers in degrees. Then use the fact that $180^\circ = \pi$ radians in order to obtain the answers in radians (as multiples of π).
- The angle whose sine is $1/2$.
 - The angle whose cosine is $1/2$.
 - The angle whose sine is $(1 \div \sqrt{2})$.
 - The angle whose cosine is $(1 \div \sqrt{2})$.
 - The angle whose sine is $(\sqrt{3} \div 2)$.
 - The angle whose cosine is $(\sqrt{3} \div 2)$.
 - The angle whose tangent is 1.
 - The angle whose tangent is $(1 \div \sqrt{3})$.
 - The angle whose tangent is $\sqrt{3}$.
- (b) Can you identify each of the angles from part (a) in these right-angled triangles?



Question 2

Calc. Book, Sec. 9.1

With your calculator in radian mode, enter $Y1 = \sin(X)$ and $Y2 = \cos(X)$. Then enter appropriate values in TBLSET so as to complete the table below.

Angle (X)/radians	Sine X	Cosine X
0		
$\pi/6$		
$\pi/3$		
$\pi/2$		
$2\pi/3$		
$5\pi/6$		
π		

In Question 1 above, what fractional expression was used instead of .86603?

Question 3**Calc. Book, Sec. 9.1**

- (a) Use your calculator to evaluate the following expressions, giving your answers to three decimal places. You may wish to set the calculator's mode to show three decimal places and to calculate in degrees.

Each pair of calculations has been chosen to illustrate some particular general feature of trigonometric functions. In each case, try some similar calculations and then state the general rule. The first one has been done for you.

- (i) The sine of 99° and the sine of 81° :

$$\sin 99^\circ = .988 \text{ and } \sin 81^\circ = .988.$$

It looks as though angles the same distance above and below 90° have the same sine. (Recall that the sine curve is symmetrical about 90° .) In general, $\sin(90^\circ + x) = \sin(90^\circ - x)$.

- (ii) The sine of 183° and the sine of 177° .
 (iii) The cosine of 10° and the cosine of -10° .
 (iv) The cosine of 9° and the cosine of 351° .
 (v) The sine of 3° and the cosine of 87° .
 (vi) The tangent of 5° and the tangent of 185° .
 (vii) $(\text{The sine of } 37^\circ) \div (\text{the cosine of } 37^\circ)$, and the tangent of 37° .
 (viii) $2 \times (\text{the sine of } 37^\circ) \times (\text{the cosine of } 37^\circ)$, and the sine of 74° .
 (ix) $(\text{The cosine of } 19^\circ)^2 - (\text{the sine of } 19^\circ)^2$, and the cosine of 38° .
 (x) $(\text{The cosine of } 19^\circ)^2 + (\text{the sine of } 19^\circ)^2$.
- (b) Now change your calculator's mode setting so that it calculates in radians. Convert each of the general rules from part (a) to the equivalent in radians and test them with some specific values. Again, the first one has been done for you.

- (i) Since $90^\circ = \pi/2$ radians,

$$\sin(90^\circ + x) = \sin(90^\circ - x), \text{ with } x \text{ in degrees,}$$

becomes

$$\sin(\pi/2 + x) = \sin(\pi/2 - x), \text{ with } x \text{ in radians.}$$

$$\text{Check: } \sin(\pi/2 + 0.1) = .995 \text{ and } \sin(\pi/2 - 0.1) = .995.$$

Question 4**Calc. Book, Sec. 9.1 and 9.2**

Use your calculator in radian mode and draw each of the following pairs of graphs. In each case, decide what the graphs have in common and how they differ.

- (a) $Y1 = \cos(X)$ and $Y2 = \cos(2X)$.
 (b) $Y1 = \sin(X) \cos(X)$ and $Y2 = \sin(2X)$.
 (c) $Y1 = \sin(X) / \cos(X)$ and $Y2 = \tan(X)$.
 (d) $Y1 = \cos(X + \pi/2)$ and $Y2 = \sin(X)$.
 (e) $Y1 = (\sin(X))^2$ and $Y2 = (\cos(X))^2$.
 (f) Enter $Y3 = Y1 + Y2$ and repeat part (e).

Question 5

Calc. Book, Sec. 9.3; Unit 9, Sec. 2

In days gone by, some guitar makers used the following 'Rule of 18' to determine the positions of the frets:

- Divide the length of the open string (i.e. between Fret 0 and the bridge) by 18, and mark off the position of Fret 1 at this distance from Fret 0.
 - Divide the length of the remaining section of string (i.e. between Fret 1 and the bridge) by 18, and mark off the position of Fret 2 at this distance from Fret 1.
 - Continue in this way until all 20 frets have been positioned.
- (a) What is the string-length ratio that is used to reduce the effective length of the string (i.e. between the last fret and the bridge) at each stage of this process? Store this value in memory location E of your calculator. Compare this with the theoretical string-length ratio needed to position the frets in order to produce an equally-tempered chromatic scale; this ratio has a value of 0.9439, and it is stored in memory location R of your calculator.
- (b) Take the length of the open string as 650 mm and use the values stored in R and E to calculate the distances of the frets from the bridge. You might do this by using an amended form of the program SCALE to store the two sets of distances in lists L1 and L2.
- (c) Compare the two sets of distances, and comment on the accuracy of the Rule of 18.

Question 6

Unit 9 Sec. 4

The frequency of the note produced by a vibrating string is inversely proportional to the length of the string. This can be expressed symbolically as $f = k \times 1/L$, where f is the frequency, L is the string length, and k is a constant. Thus, halving the length of the string will double the frequency of the note that the string produces.

The value of the constant k for a particular string depends on two factors: the weight (or gauge) of the string (measured as mass per metre), and the tension of the string (a measure of how tightly it is stretched). If you measure the length of the string in metres, then $k = \frac{1}{2}\sqrt{T/M}$, where M is the mass per unit length of the string, measured in kilograms per metre, and T is the tension, measured in newtons. Thus the full relationship between frequency, length, tension and mass per metre is

$$f = \frac{\sqrt{T/M}}{2L}.$$

So, for a particular string of fixed length, the tension would need to be quadrupled in order to double the frequency.

- (a) Rearrange this equation to make T the subject.
- (b) The middle C string of a particular piano is 64 cm long and weighs 3.8 g.
- (i) What is its mass per unit length, in kg/m?
 - (ii) To what tension, in newtons, must it be stretched in order to produce a note of the required frequency for middle C (256 Hz)?

Question 7

Unit 9 Sec. 4

Ron, an MU120 student and a keen long-distance cyclist, wants to develop a formula like Naismith's rule to estimate the times of cycle journeys.

He frequently cycles from A over a hill to H, a distance of 10 km. Using an OS map, he decides to mark intermediate positions B, C, D, E, F and G where the road crosses contour lines on the map. He measures the distance of each point from the start and records the height above sea level. These data are recorded below.

- (a) Input these data as lists in your calculator and plot the points as a line graph. What implicit assumptions are made in this representation?
- (b) Ron decides to use Naismith's rule to work out how long the journey would take if he were walking. Naismith's rule states that

$$t = \frac{d}{5} + \frac{h}{600},$$

where t is the time in hours taken to walk a distance d km involving climbing a height h m. Ron would prefer to work in minutes (T) rather than hours (t). Write down an equation linking T and t . Multiply every term in Naismith's rule by 60 to obtain an equation $T = \dots$

- (c) Use this rule to estimate the time taken to complete each section of the journey. Then fill in columns (4)–(6) of the table.

(1) Point on journey	(2) Distance from start/km	(3) Height above sea level/m	(4) Distance from previous point/km	(5) Height above previous point/m	(6) Time taken from previous point/min	(7) Mean time taken by Ron/min
A	0	100				
B	2	100				
C	4	120				
D	5	120				
E	6	160				
F	7	180				
G	8	160				
H	10	140				

- (d) Spend a few moments thinking about how you would expect the times in column (6) to differ if Ron were on his bike, rather than walking.

Whereas (according to Naismith's rule) a walker does not go faster when going downhill, Ron knows that on his cycle he travels faster downhill than when on the flat. He wonders how Naismith's rule will need to be changed to accommodate this. What do you think?

Ron decides that Naismith's rule:

$$T = 12d + \frac{h}{10}$$

will change to Ron's rule:

$$T = Pd + \frac{h}{Q} - \frac{h'}{R}$$

where h' is the height descended in m, and P , Q and R are constants which he needs to work out.

- (e) Ron now completes the journey by bike several times, on each occasion recording the times taken in minutes for the individual sections of the journey. The mean times, rounded to the nearest minute, are 0, 12, 16, 6, 14, 10, 2, and 8, respectively. Enter these times in column (7).
- (f) Consider the parts of the journey where there is no change in height, (i.e. $h = 0$ and $h' = 0$). Write down the corresponding simplified version of Ron's rule and manipulate the equation so as to make P the subject. Use appropriate data from the table to calculate the value of P .

Write down a full version of Ron's rule, without using P .

- (g) Now consider the parts of the journey where there is an increase in height but no decrease (i.e. $h' = 0$). Write down the corresponding simplified version of Ron's rule and manipulate the equation to make Q the subject. Use appropriate data from the table to calculate the value of Q .

Write down a full version of Ron's rule, without using P or Q .

- (h) Use a similar method to calculate the value of R and write down a full version of Ron's rule, without using P , Q or R .
- (i) Ron now sees that, because of the values of Q and R , he can simplify the rule at a stroke. There is no need to think of h as the height climbed and h' as the height descended. They can both be replaced by a new variable (say, w) which represents the vertical displacement in m. Then $w = h - h'$. Write down the final simplified version of Ron's rule and test it for various parts of the journey.
- (j) Use Ron's rule to calculate the time taken for the journey back from H to A. Why do you think there is something wrong with Ron's rule?

Question 8

Unit 9 Sec. 4

A Pythagorean triple consists of three whole numbers which will satisfy Pythagoras' theorem. One such triple is $[3, 4, 5]$ because $3^2 + 4^2 = 5^2$.

- (a) Complete these Pythagorean triples:

$$[5, 12, \dots], \quad [9, \dots, 15].$$

- (b) There are several formulas for generating such triples, one of which is

$$[n, (n^2 - 1)/2, (n^2 + 1)/2].$$

This works when n is any positive odd number over 2. Use this formula and your calculator to find the triples generated when

$$(i) \ n = 5, \quad (ii) \ n = 9, \quad (iii) \ n = 17, \quad (iv) \ n = 21.$$

- (c) Work through the following steps to prove that this formula works. A Pythagorean triple such as $[A, B, C]$ must satisfy the formula:

$$A^2 + B^2 = C^2.$$

Now let $A = n$, $B = (n^2 - 1)/2$ and $C = (n^2 + 1)/2$.

- (i) Work out a value for $A^2 + B^2$, and simplify it as much as possible.
- (ii) Work out separately and simplify the value of C^2 .
- (iii) Try to show that the expression in part (i) is the same as the expression in part (ii). You will then have proved that $[n, (n^2 - 1)/2, (n^2 + 1)/2]$ is a Pythagorean triple.

Unit 6

1

- (a) The features are:
- 168829 tumulus
 - 147844 reservoir
 - 136880 Madwoman's Stones
 - 150829 church with tower.
- (b) The grid references are:
- Crook Hill 183868
 - Druid's Stone 134874
 - Blue John Cavern 131831
 - railway station 181832.

Remember that six-figure grid references specify the south-western corner of the 100 m square containing the feature. Do not worry if your references differ slightly from these.

2 The grid squares have sides of length 100 kilometres, or 100 000 metres. At a scale of 1 : 50 000, the map will be a square with sides of length $100\,000/50\,000 = 2$ metres.

3

- (a) Multiply the map distance in cm by 250 000, divide by 100 000 to get the ground distance in km. Round to one decimal place.

Feature	Map distance/ cm	Ground distance/ km
Guide post	5.0	1.3
Cairn	3.3	0.8
Madwoman's Stones	5.0	1.3
Shooting cabin	5.7	1.4

- (b) Guide post:

$$\begin{aligned} &\sqrt{(160 - 147)^2 + (873 - 874)^2} \\ &= 13.04 \text{ units (of 100 metres)} \\ &= 1.304 \text{ km.} \end{aligned}$$

The previous calculator entry can be edited to produce the calculations for the next three locations.

Cairn:

$$\begin{aligned} &\sqrt{(149 - 147)^2 + (882 - 874)^2} \\ &= 8.25 \text{ units} = 0.825 \text{ km.} \end{aligned}$$

Madwoman's Stones:

$$\begin{aligned} &\sqrt{(136 - 147)^2 + (880 - 874)^2} \\ &= 12.53 \text{ units} = 1.253 \text{ km.} \end{aligned}$$

Shooting cabin:

$$\begin{aligned} &\sqrt{(142 - 147)^2 + (887 - 874)^2} \\ &= 13.93 \text{ units} = 1.393 \text{ km.} \end{aligned}$$

4 The difference between magnetic north and grid north is decreasing by about $\frac{1}{2} \div 3 = \frac{1}{6}^\circ$ per year. If this rate remains constant, the difference will decrease by $10 \times \frac{1}{6} = \frac{10}{6} \simeq 1.7^\circ$ between 1994 and 2004. So the direction of magnetic north in 2004 will be about $5 - 1.7 = 3.3^\circ$ west of grid north.

5 Looking back from Win Hill, the compass bearing of the walker's position is 180° greater than the bearing she measured. So the bearing from Win Hill is $83 + 180 = 263^\circ$. Similarly, the compass bearing looking back from the chimney is $119 + 180 = 299^\circ$.

To convert to grid bearings, subtract 5° from each bearing to compensate for the difference between grid north and magnetic north. Using a protractor and your OS map extract, draw lines at grid bearings of $263 - 5 = 258^\circ$ and $299 - 5 = 294^\circ$ from Win Hill and the chimney, respectively. You should find that the lines cross near Mam Farm, where the footpath meets the old road from Castleton (grid reference 132839). This is where the walker is standing.

6 Drawing up a table is a useful way of collecting together the relevant details. The following table lists approximate distances on the walk from one point to the next and the corresponding heights ascended, found from the OS map extract. Naismith's rule has been used to estimate the times from one point to the next, which are given in the final column.

Route	Distance/ km	Height ascended/m	Time/ hours
Car park	0	0	0
Hollins Cross	2.0	97	0.56
Backtor Bridge	1.4	0	0.28
Beginning of Harden Clough	1.6	22	0.36
Car park	2.3	223	0.83
Total	7.3	342	2.03

Naismith's rule requires knowledge of the distance and the total height ascended for each leg of the walk. The time for each leg can then be calculated using the formula:

time (hours) = $\frac{\text{distance (km)}}{5} + \frac{\text{height ascended (m)}}{600}$

For example, on the first leg to Hollins Cross a walker must ascend a total height of about 97 metres to reach Mam Tor (after that it is downhill to Hollins Cross), over a total distance of 2 km. Naismith's rule gives the time for this section as

time (hours) = $\frac{2}{5} + \frac{97}{600} \simeq 0.4 + 0.16$
= 0.56, or about 34 minutes.

Using Naismith's rule on each stage of the walk and adding the results gives an estimate of just over 2 hours to cover 7.3 km. You should get the same result if you use Naismith's rule on the entire walk with a figure of 342 metres for the total height ascended.

7 The height of the cairn at 147874 is just over 440 metres. The table gives estimates of the height, change in height, distance and gradient data.

Feature	Height/ m	Change in height/ m	Distance/ m	Average gradient
Guide post	300	-150	1250	-0.1
Cairn	440	0	825	0
Madwoman's Stones	565	125	1250	0.1
Shooting cabin	400	-40	1425	-0.03

Note distances are given to 3 significant figures here and so differ slightly from those given in Solution 3.

8 3 miles is equal to 3×1760 yards. This is $3 \times 1760 \times 3$ feet, or $3 \times 1760 \times 3 \times 12$ inches = 190 080. So 1 inch on the map represents 190 080 inches on the ground. Hence the map scale is 1 : 190 080.

1 inch represent 3 miles, or $3 \times 1.61 = 4.83$ km. So an area of 1 square inch (1 inch $\times$ 1 inch) on the map represents an area of $4.83 \times 4.83 = 23.3289 \text{ km}^2$ on the ground.

A hectare is an area of $10\,000 \text{ m}^2$, which is 0.01 km^2 (because $1 \text{ km}^2 = 1\,000\,000 \text{ m}^2$). So a map area of 1 square inch represents an area of $23.3289/0.01 = 2332.89 \text{ ha} = 2333 \text{ ha}$ (to nearest hectare).

9 The area scaling factor is equal to the square of the map scale.

- (a) An area of 1 cm^2 on the map represents an area of 50 ha on the ground. To find the area scaling factor, find the number of square centimetres in 50 hectares.

Now $1 \text{ ha} = 10\,000 \text{ m}^2$, and $1 \text{ m}^2 = 100 \times 100 \text{ cm}^2 = 10\,000 \text{ cm}^2$. So $50 \text{ ha} = 50 \times 10\,000 \text{ m}^2 = 500\,000 \text{ m}^2$, which is equivalent to $500\,000 \times 10\,000 = 5\,000\,000\,000 \text{ cm}^2$, or $5 \times 10^9 \text{ cm}^2$.

Therefore, the area scale of the map is 1 : 5 000 000 000, or $1 : 5 \times 10^9$. This is the square of the map scale, so take the square root:

$\sqrt{5 \times 10^9} \simeq 7.0711 \times 10^4 = 70\,711$.

Thus the map scale $\simeq 1 : 70\,711$.

- (b) If the map scale were 1 : 100 000, then 1 cm^2 on the map would represent $100\,000^2 = 10\,000\,000\,000 = 10^{10} \text{ cm}^2$ on the ground. Now $1 \text{ ha} = 10\,000 \text{ m}^2$ and since $1 \text{ m}^2 = 10\,000 \text{ cm}^2$, there are $10\,000 \times 10\,000 = 100\,000\,000 = 10^8 \text{ cm}^2$ in 1 ha.

Thus 10^{10} cm^2 corresponds to $10^{10}/10^8 \text{ ha} = 10\,000\,000\,000/100\,000\,000 \text{ ha} = 100 \text{ ha}$. So 1 cm^2 on the map represents 100 ha at this scale.

Notice that reducing the map scale from 1 : 70 711 to 1 : 100 000, corresponds to reducing the scale by a factor of $\sqrt{2}$, since $70\,711 \times \sqrt{2} \simeq 70\,711 \times 1.41421 \simeq 100\,000$. The effect is to increase the area represented by 1 cm^2 on the map by a factor of 2, since $\sqrt{2} \times \sqrt{2} = 2$.

10 The first task is to collect from the map, the data with which to draw the profile. Two sets of data are required for each point: height in metres (from the contours) and distance from the start point (the cairn at 147874). The distances are measured in millimetres on the map, and then converted to metres using the scale of the map. One millimetre on the map is the equivalent of 25 metres on the ground.

Opposite are the data collected by one person. However, it is very difficult to be accurate on these sorts of distances, so do not be surprised if your data differ slightly.

Height/m	440	430	420	410	400	390
Distance/mm	1	5	7.5	9	10	11
Distance/m	25	125	188	225	250	275

380	370	360	350	340	340	350	360	370
12	13	14.5	15	15.2	16	17	18	19
300	325	363	375	380	400	425	450	475

380	390	400	410	420	430	440	442
20	21	22	24	26	28.5	32	33
500	525	550	600	650	713	800	825

Now to draw the profile. Choose distance in metres as the x variable and enter these data into list L1. Then enter the heights in list L2.

Next an appropriate window is needed. The ' x ' data, the distances in metres, go from 0 to 825 metres, and the ' y ' data, the heights, go from 340 to 442 m. So suitable first six window settings are 0, 850, 100, 330, 450, 10. Now set up a statistical plot, choosing a line graph of the data in L1 and L2.

Pressing GRAPH gives the profile, which should look something like this.

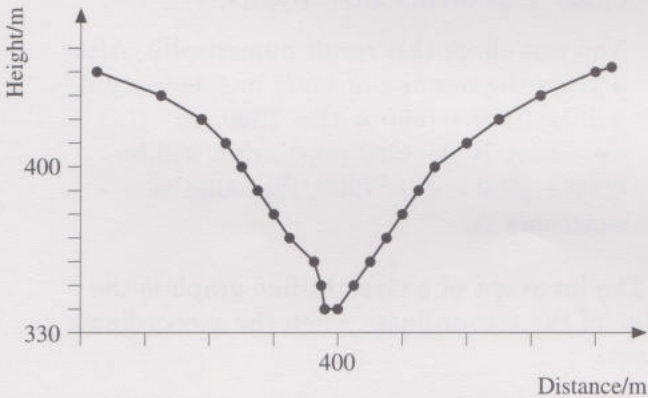


Figure 2

The gradient is close to zero at both cairns and is zero in the bed of the brook which is at a height of 340 m at a distance between 380 and 400 m from the first cairn.

11

- (a) The grid reference of a point has six figures. The first three give the 'easting' of the point, and the last three give the 'northing'. So the data to be entered to lists L1 and L2 are

Eastings (L1)	197	197	190	183	187	197
Northings (L2)	849	856	860	850	851	849

The easting values are all within the range 183 to 197, and the northings are between 849 and 860. A small extension at each end of these ranges will place the graph in the centre of the screen, so set the first six window values to 180, 200, 1, 845, 865, 1.

Display the graph by selecting, from the STAT PLOT menu, a line graph, L1, L2, and the cross. Your graph should be similar to this.

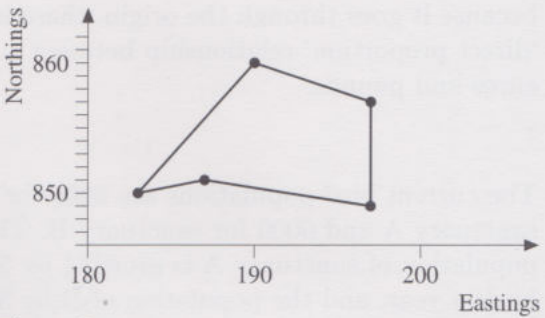


Figure 3

- (b) The method is explained in the *Calculator Book*, Section 6.2.
- The table below shows the values displayed in the calculator's lists at the end of the process.

L1	L2	L3	L4	L5
197.0	849.0	197.0	856.0	.7
197.0	856.0	190.0	860.0	.8
190.0	860.0	183.0	850.0	1.2
183.0	850.0	187.0	851.0	.4
187.0	851.0	197.0	849.0	1.0

The total distance for the walk is the sum of the values in L5—about 4.1 km.

Unit 7

1

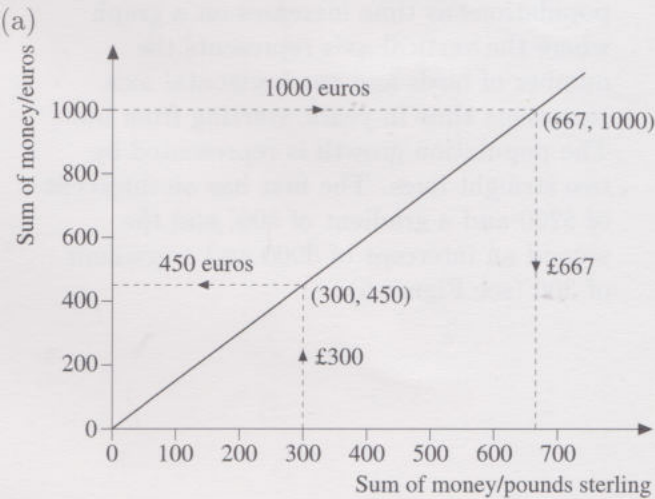


Figure 4

The graph is drawn so that it passes through the two fixed points (300, 450) and (667, 1000). Notice also that the graph passes through the origin (0,0) because zero euros is equivalent to zero pounds.

- (b) From the graph you should find that £400 is equivalent to about 600 euros, and that 750 euros is equivalent to about £500.
- (c) The graph is linear (i.e. a straight line) and, because it goes through the origin, there is a 'direct proportion' relationship between euros and pounds.

2

- (a) The current bird populations are 5200 for sanctuary A and 6000 for sanctuary B. The population of sanctuary A is growing by 500 birds a year, and the population of B by 300 birds per year. So, for example, the population of sanctuary A is $5200 + 500 = 5700$ after the first year, $5200 + (500 \times 2) = 6200$ after the second year, $5200 + (500 \times 3) = 6700$ after the third, and so on. The bird population of sanctuary B grows in a similar way. So the general word formula for predicting bird populations is

bird population after a number of years
= current number of birds +
(growth per year $\times$ number of years).

For sanctuary A, this gives

bird population
= $5200 + (500 \times \text{number of years})$,

and for sanctuary B

bird population
= $6000 + (300 \times \text{number of years})$.

- (b) You can plot the growth of the bird populations as time increases on a graph where the vertical axis represents the number of birds and the horizontal axis represents time in years, starting from now. The population growth is represented by two straight lines. The first has an intercept of 5200 and a gradient of 500, and the second an intercept of 6000 and a gradient of 300 (see Figure 5).

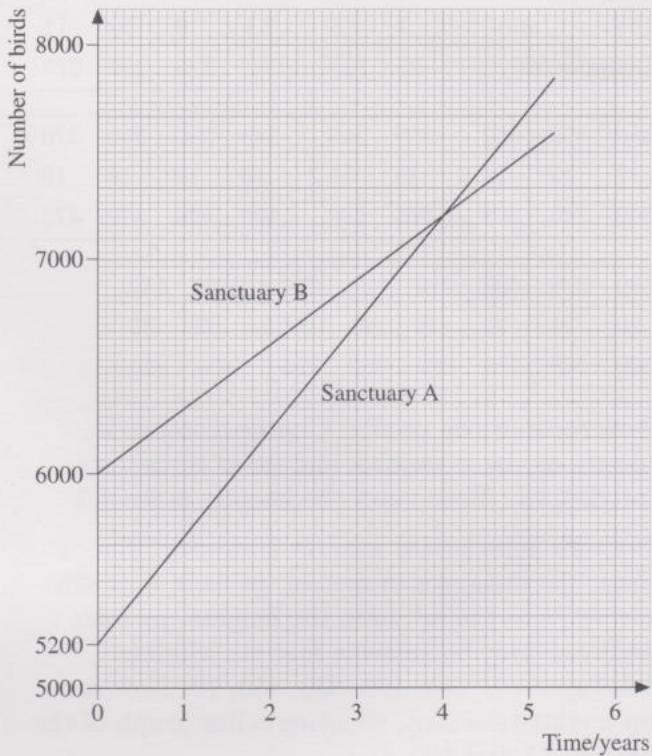


Figure 5

The populations will be equal when the lines cross. This occurs after 4 years.

You can check this result numerically. After 4 years the number of birds in sanctuary A will be $5200 + (500 \times 4) = 7200$. In sanctuary B the bird population will be $6000 + (300 \times 4) = 7200$, the same as sanctuary A.

3 The intercept of a straight-line graph is the value of the y -coordinate when the x -coordinate is zero.

The gradient is the slope of the graph. It is defined as

$$\text{gradient} = \frac{\text{increase in } y\text{-coordinate}}{\text{increase in } x\text{-coordinate}}.$$

If the y -coordinate decreases as the x -coordinate increases, the change in y is taken as a *negative* increase.

- (a) The y -axis is where $x = 0$. So the intercept is where the straight line crosses the y -axis, namely 1.

The y -coordinate increases from 0 to 2 as the x -coordinate increases from -2 to 2, so the gradient is positive and equal to

$$\frac{(2 - 0)}{(2 - (-2))} = \frac{2}{4} = 0.5.$$

- (b) The intercept is where the straight line crosses the y -axis, namely 1.5.

The value of the y -coordinate stays at 1.5 whatever the value of the x -coordinate. Increasing x results in a zero increase in the value of y , so the gradient of the horizontal straight line is zero.

- (c) The intercept is where the straight line crosses the y -axis, namely 2.

As the x -coordinate increases from 0 to 2, the y -coordinate changes from 2 to -1, an 'increase' of -3. So the gradient of the line is $-\frac{3}{2} = -1.5$.

- (d) The straight line passes through the origin of the graph, so the intercept is zero.

The graph also passes through the point (3, 2), so the gradient is $\frac{2}{3}$, or 0.67.

- (e) In this graph the axes meet not at the origin but at (3, 0), so the intercept (the value of the y -coordinate when the x -coordinate is zero) cannot be read off directly.

The graph is redrawn below with the x -axis extended back to zero. You can see that the straight line passes through the origin, so the intercept is zero.

The gradient is $2/3$.

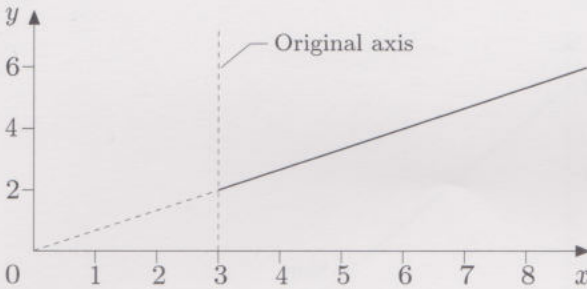


Figure 6

(The relationship between y and x on this graph is the same as that in (d). In fact, (d) and (e) are the same graph drawn on different axes.)

- (f) In this graph the x -axis starts at 5 and the y -axis starts at 2, so you must be careful about finding the intercept. The graph is redrawn below with the x -axis extended to zero, clearly showing the intercept as 7.

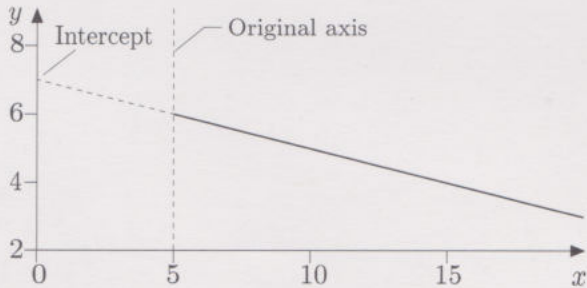


Figure 7

As the x -coordinate increases from 5 to 15, the y -coordinate decreases from 6 to 4. So the gradient is $-2/10 = -0.2$.

4

x -coordinate	y -coordinate	Coordinate pair
-2.5	6.25	(-2.5, 6.25)
-1.5	2.25	(-1.5, 2.25)
-0.5	0.25	(-0.5, 0.25)
0	0	(0, 0)
1	1	(1, 1)
2	4	(2, 4)
3	9	(3, 9)

The graph drawn from these data is shown in Figure 8. The value of the y -coordinate is positive for both positive and negative values of the x -coordinate (because the square of a negative number is always a positive number). For negative values of the x -coordinate, the slope or gradient of the graph is negative (sloping down from left to right), becoming less steep as the x -coordinate gets closer to zero. At the origin of the graph, the point (0, 0), the slope is zero. For positive values of the x -coordinate, the slope is positive (sloping up from left to right), becoming steeper as the x -coordinate increases.

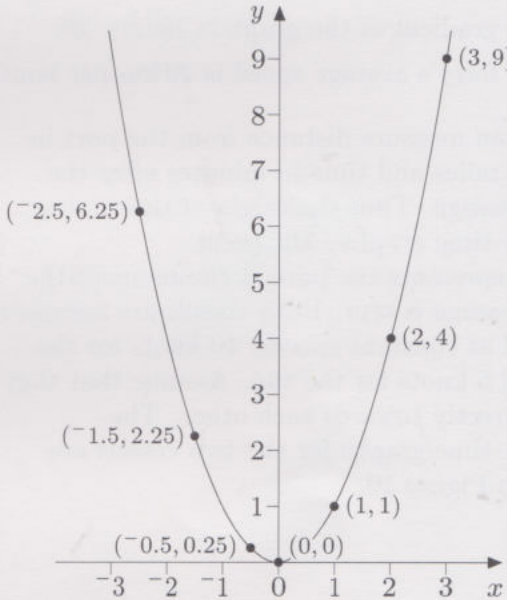


Figure 8

5

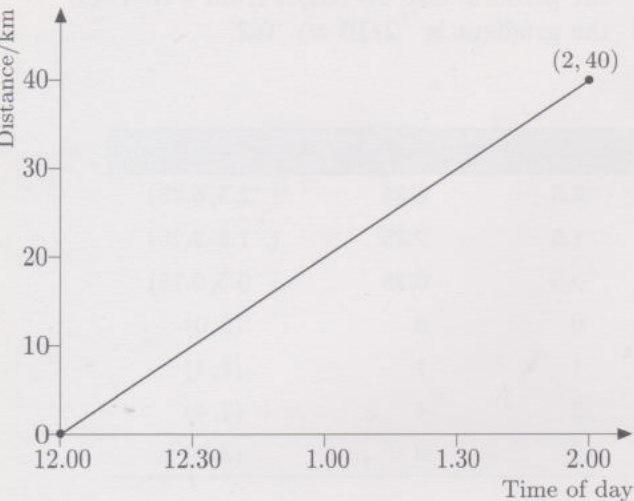


Figure 9

- The distance–time graph is shown above. The straight line starts at the origin of the graph, representing the ferry’s departure from the English port at 12 noon, and finishes at the point (2, 40), representing its arrival at the French port at 2.00 pm (GMT).
- (a) The ferry has gone 15 km by 12.45 pm, and 35 km by 1.45 pm.
 - (b) By 12.15 pm it is 5 km away from the English port, and by 1.10 pm it is about 23 km away.
 - (c) The gradient of the graph is $40/2 = 20$.
 - (d) The ferry’s average speed is 20 km per hour.

6 You can measure distance from the port in nautical miles and time in minutes after the radio message. Thus the origin of the distance–time graphs—the point (0, 0)—represents the port at the moment the radio message is sent. Both vessels are assumed to travel at constant speeds, 10 knots for the ship and 5 knots for the tug. Assume that they travel directly towards each other. The distance–time graphs for the two vessels are shown in Figure 10.

At time 0 the ship is 10 nautical miles from port, so the point (0, 10) represents the position of the ship when it sent the message. If it continued to travel towards the port at a steady speed of 10 knots, it would arrive there after 1 hour. The ship’s predicted motion is represented by the line joining (0, 10) and (60, 0).

The tug starts from port 30 minutes after the message is sent, represented by the point (30, 0). If it travels at 5 knots for 30 minutes, it will have gone 2.5 nautical miles. The tug’s predicted motion is represented by the line joining (30, 0) and (60, 2.5).

The distance–time graph model predicts that the tug and the ship should meet (where the lines cross) about 50 minutes after the ship radioed. (There is no information about what happens when the ship and the tug meet, so the broken lines indicate what their motion would have been if their speeds remained unchanged.)

Since the tug would have to accelerate up to 5 knots, it might not average as much as 5 knots. So there is probably no need to keep a look out for the ship until about 50 minutes after the radio message.

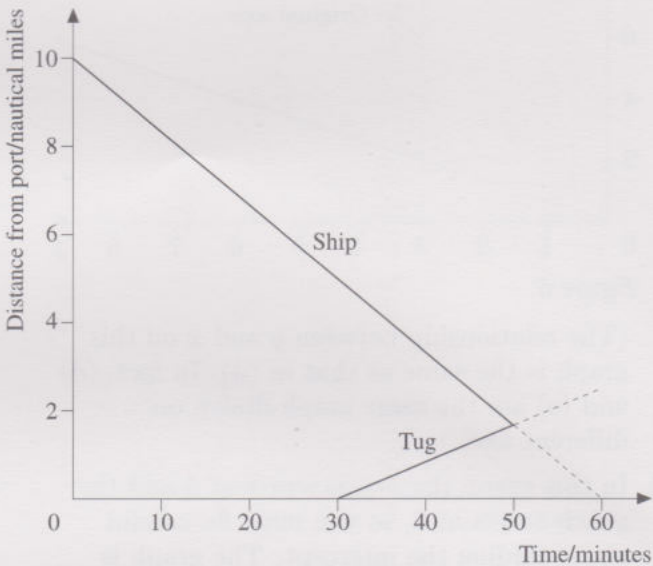


Figure 10

7

- (a) The vertical axis of the graph represents the distance in kilometres walked *from the start* (add together the distances for the relevant legs of the walk), and the horizontal axis represents the time in hours, measured *from the start* (add together the times for the relevant legs of the walk). So the start of the walk is represented by the point (0,0) on the graph, and the finish, 2.03 hours later after walking 7.3 km, is represented by the point (2.03, 7.3).

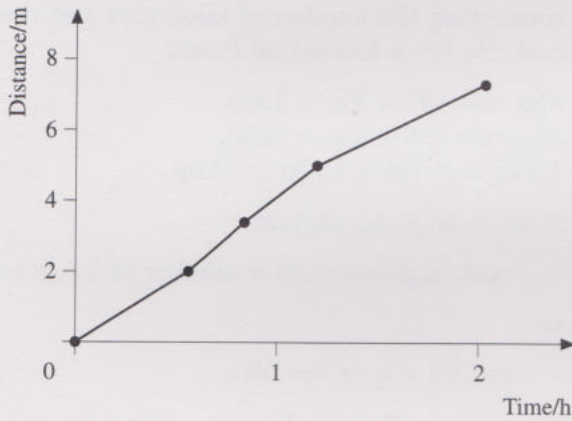
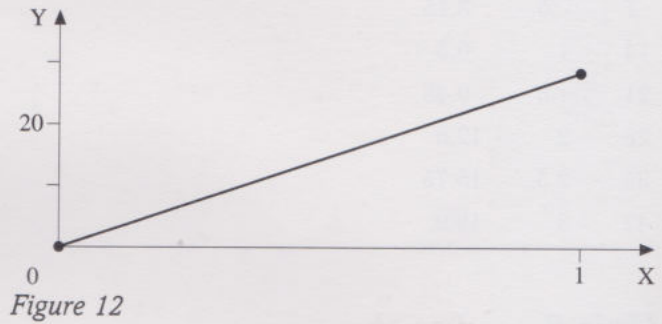


Figure 11

- (b) You can see from the slopes of the different parts of the graph that the predicted speed is greatest (the graph has the greatest slope) on the second leg: downhill between Hollins Cross and Backtor Bridge. The speed is least (the graph has the smallest slope) on the final (uphill) leg along Harden Clough to the car park.
- (c) The average speeds are about 3.6, 5.0, 4.4 (or 4.5, if you did not round intermediate answers) and 2.8 km per hour.
- (d) The average speed for the entire walk is about $7.3 \div 2.03 \approx 3.6$ km per hour.

8

- (a) 28.4 ml.
- (b) Enter the function $Y1 = 28.4X$ and obtain a graph like that below.



- (c) (i) 8 fl.oz = 227.2 ml.
(ii) 200 ml = 7.04 fl.oz.
- (d) Your answers may not be exactly the same as in part (c), because your choice of window may mean that the pixels do not correspond exactly to 8 fl.oz or 200 ml.

9

- (a) Writing y for the distance in kilometres and x for the time in hours, a suitable formula is $y = 95x$ or $Y1 = 95X$.
- (b) Set the table difference to 0.5.

X	Y1	X	Y1
0	0	2.5	237.5
0.5	47.5	3.0	285.0
1.0	95.0	3.5	332.5
1.5	142.5	4.0	380.0
2.0	190.0	4.5	427.5

- (c) After two hours you would be 190 km from your starting point.
- (d) By setting the table minimum to 1.5 and the table difference to 0.1, you can see that you will pass the first service area after just over 1.8 hours or about 1 hour 50 min, and the second after just over 2.3 hours or about 2 hours 20 min.

10 Enter the functions $Y1 = X/14$, $Y2 = .45X$. Set the table difference to 7, because there are 7 lbs in a half-stone.

X	Y1	Y2
0	0	0
7	.5	3.15
14	1	6.3
21	1.5	9.45
28	2	12.6
35	2.5	15.75
42	3	18.9

Unit 8

1 Here are some points to consider when you are manipulating positive and/or negative quantities.

- Numbers or symbols which have no sign in front of them are positive.
- Decide whether the operation you are being asked to do is addition or subtraction, or multiplication or division.
- If it is addition or subtraction, then follow the signs and add or subtract as you usually do.
- If it is multiplication or division, then use the rule of signs, which says:

If the two signs of the quantities to be multiplied or divided are the same, then the answer will have a + sign. If they are different, the answer will have a - sign.
- A minus sign outside a bracket changes all the signs inside the bracket.

Here are the answers:

- (a) 6 (b) -2 (c) 4a
(d) becomes $4 + 3 = 7$ (e) -2 (f) 8
(g) becomes $121p + 43p + 46p = 210p$
(h) -56
(i) $+3 \times +4 \times -9 = +12 \times -9 = -108$
(j) 2304 (k) -144 (l) -0.5 (m) 4q
(n) becomes $-4 + (+12) = 8$.

2

- (a) The first step in finding a formula is to decide which variables are to be represented by letters. It is tempting to think 'b for bananas', but is it the number of bananas, the price of bananas, the weight of bananas, or something different? You know that the price is 95p per kg and it is the number of kilograms that varies. So let n (or b or W or whatever) stand for the number of kilograms of bananas. Let c stand for the total cost in pence.

The next step is to try to discover a pattern connecting the number of kilograms and the cost. So try a few actual cases:

- 3 kg cost $(95 \times 3)p = 195p$,
5 kg cost $(95 \times 5)p = 325p$,
1.5 kg cost $(95 \times 1.5)p = 97.5p$.

It looks as if the pattern is

$$\text{cost in pence} = 95 \times \text{number of kilograms,}$$

or

$$c = 95 \times n \text{ or } c = 95n.$$

- (b) 7p will be added to the cost, giving a formula of $c = 95n + 7$. Note that the '7' stays as '7' irrespective of the weight of bananas purchased. (Assume only one bag is needed.)

Notice that c is the cost in pence. If C stands for the cost in £s, then
 $C = (95n + 7)/100$.

- (c) The cost of 2.6 kg in a bag is
 $£((95 \times 2.6) + 7)/100$, which equals £2.54.

3 A formula is needed which gives the relationship between number of units (abbreviate to u) and the amount of the bill in £s (call it b).

The cost of the calls alone will be number of units multiplied by 2.4p, but to get the answer in £s this must be divided by 100. So the cost of calls in £s = (number of units $\times$ 2.4)/100, or $0.024u$.

But the total bill equals the cost of the calls plus the cost of the standing charge, so

$$b = 0.024u + 15.30.$$

4

- (a) When $q = 2$, $p = 3 \times 2 + 2 = 8$.
 (b) When $q = 1.74$, $p = 3 \times 1.74 + 2 = 7.22$.
 (c) When $q = -1.5$, $p = 3 \times -1.5 + 2 = -2.5$.

5 The dimensions of the ark, using the standard conversion of the cubit, were 138 metres by 23 metres by 13.8 metres. This means that it was small by the standards of a modern cruise ship.

6 $5; \frac{64}{15}$ (or $4\frac{4}{15}$).

7

- (a) $5N$; $30z$; $40y$.
 (b) $-5y$; $22x$; $2p + 9p^2$.

8

- (a) $4y + 4x$ or $4(y + x)$
 (b) $5p + 2r$ or $2r + 5p$
 (c) $12\clubsuit + 14\spadesuit$. (N.B. Symbols don't have to be letters.)
 (d) $4k - 13k + s$ or $-9k + s$ (or $s - 9k$)
 (e) $2A^2 + 3A^2 = 5A^2$
 (f) ab means the same as ba because the order of symbols within a term does not matter; so the first and third terms can be combined to give $1 \times ab$ or just ab . Similarly the second and fourth terms can be combined to give $4ab^2$, and the final answer is $ab + 4ab^2$.

9 Anything, number or symbol, outside the bracket multiplies everything inside the bracket.

- (a) $6x - 2$ (b) $4k + 20$ (c) $2pq + p^2$
 (d) $2ma - 2mb + 2mc$ (e) $2b^2 + 2b^3 - 2b^4$
 (f) $-3x - 6y + 3z$ (a minus sign before the term immediately outside the bracket multiplies everything by -1 , so every sign inside the bracket is changed)
 (g) $2r - 3r^2$.

10 Using the method in Example 1 of Unit 8:

- (a) $(2 + x)(5 - y) = (2 + x) \times 5 + (2 + x) \times -y$
 $= 10 + 5x - 2y - xy$
 (b) $(p + q)(x + y) = (p + q)x + (p + q)y$
 $= px + qx + py + qy$
 (c) $(3 - y)(4 + y) = 4(3 - y) + y(3 - y)$
 $= 12 - 4y + 3y - y^2 = 12 - y - y^2$
 (d) $(5f - 3)(2f - 1) = 2f(5f - 3) - 1(5f - 3)$
 $= 10f^2 - 6f - 5f + 3 = 10f^2 - 11f + 3$.

11

- (a) Yes ($4 \times 4 = 16$).
 (b) Yes (the order of symbols may be changed).
 (c) No (p does not equal p^2 nor does q^2 equal q).
 (d) No (powers do not match the original expression).
 (e) Yes (it multiplies out correctly).
 (f) Yes (32 divided by 2 is 16, and q^2 divided by q is q).

12

- (a) $Y1(3) = 9.7$, $Y1(13.72) = 52.58$,
 $Y1(-101.5) = -408.3$.
 (b) You will need to use different window settings for each part of the question but you should be able to produce similar answers to those in part (a).

13

- (a) Step 1: multiply by 4.2 (to get $4.2q$)
 Step 2: subtract 1.7 (to get $4.2q - 1.7$)
 (b) To rearrange the formula, undo the steps in the reverse order.
 Step 1: add 1.7 (to get $p + 1.7 = 4.2q$)
 Step 2: divide by 4.2 (to get $(p + 1.7)/4.2 = q$)
 (c) So $q = (p + 1.7)/4.2$.

14

- (a) Step 1: divide by 3 (to get $x/3$)
 Step 2: add 9 (to get $x/3 + 9$)
 To rearrange the formula, undo the steps in the reverse order.
 Step 1: subtract 9 (to get $y - 9 = x/3$)
 Step 2: multiply by 3 (to get $3(y - 9) = x$)
 (b) Step 1: multiply by 12 (to get $12x$)
 Step 2: add 5 (to get $5 + 12x$)

To rearrange the formula, undo the steps in the reverse order.

Step 1: subtract 5 (to get $y - 5 = 12x$)

Step 2: divide by 12 (to get $(y - 5)/12 = x$)

(c) Step 1: multiply by -2 (to get $-2x$)

Step 2: add 12 (to get $12 - 2x$)

To rearrange the formula, undo the steps in the reverse order.

Step 1: subtract 12 (to get $y - 12 = -2x$)

Step 2: divide by -2 (to get $(y - 12)/-2 = x$ which is the same as $(12 - y)/2 = x$).

15

(a) To get $2x + 5$, x is multiplied by 2 and then 5 is added, in that order.

Since $2x + 5 = 8$, apply the reverse operations in the reverse order to the equation so as to get the value of x .

The left-hand side gives x and the right-hand side becomes 8 subtract 5, which gives 3, and 3 divided by 2 gives 1.5. So $x = 1.5$.

(b) If $x = 1.5$, $2x + 5$ becomes 2 times 1.5, plus 5, which equals 3 plus 5, or 8. So $x = 1.5$ is correct.

16

(a) $x = 1.5$ (b) $x = -1.5$

(c) Before solving, simplify by 'collecting like terms' so $4x + 2 = 20$. Then $x = 4.5$.

(d) Simplify to give $5x - 10 = 8$. Then $x = 3.6$.

17 Let Gretel's age be x years. So Hansel's age is $x + 6$. In two years' time, Hansel will be $x + 8$ and Gretel will be $x + 2$. Hansel will then be three times as old as Gretel, therefore $x + 8 = 3(x + 2)$.

Expanding the bracket gives $x + 8 = 3x + 6$, hence $2 = 2x$. So Gretel is one year old. (Check: Hansel is seven; in two years' time he will be nine and Gretel will be three.)

18

(a) When $x = 2.25$, $y = 9.328$ (to 3 d.p.).

(b) When $y = 10$, x can have three different values: 2.28, -0.66 and -4.62 (all correct to 2 d.p.).

19

(a) Seventeen fuschias at $\pounds f$ each plus $\pounds 2.36$ equals $\pounds 11.71$, so the equation is $17f + 2.36 = 11.71$.

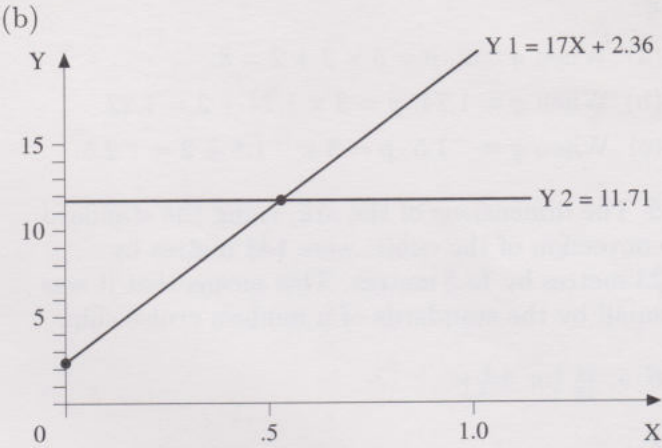


Figure 13

- (c) If $Y1 = 17X + 2.36$ and $Y2 = 11.71$, the two graphs will meet where $Y1 = Y2$, i.e. where $17X + 2.36 = 11.71$. The value of X at this point will be the solution of the equation.
- (d) At this point, $x = 0.55$. So each fuschia cutting cost 55p.

20

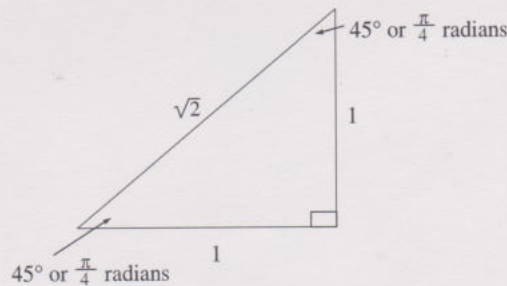
- (a) The graph peaks at $y = 6.25$ and $x = 2.5$. So the firework reaches a maximum height of 6.25 m after 2.5 seconds.
- (b) The required function would be $y = 6$.
- (c) The two graphs cut twice, at $x = 2$ and $x = 3$, indicating that the firework is at a height of 6 m after 2 seconds on its way up and after 3 seconds on its way down.

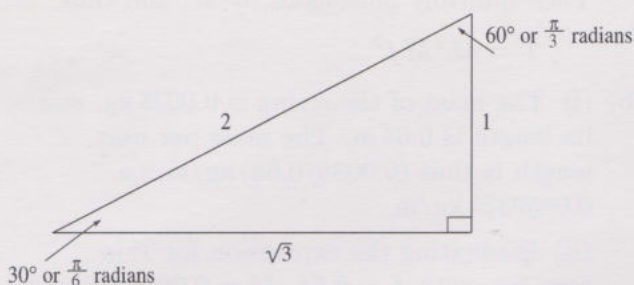
Unit 9

1

- (a)
- | | | | |
|-------|------------------------------------|--------|------------------------------------|
| (i) | $30^\circ = \frac{\pi}{6}$ radians | (ii) | $60^\circ = \frac{\pi}{3}$ radians |
| (iii) | $45^\circ = \frac{\pi}{4}$ radians | (iv) | $45^\circ = \frac{\pi}{4}$ radians |
| (v) | $60^\circ = \frac{\pi}{3}$ radians | (vi) | $30^\circ = \frac{\pi}{6}$ radians |
| (vii) | $45^\circ = \frac{\pi}{4}$ radians | (viii) | $30^\circ = \frac{\pi}{6}$ radians |
| (ix) | $60^\circ = \frac{\pi}{3}$ radians | | |

(b)





2 In TBLSET you need to enter TblStart = 0 and $\Delta\text{Tbl} = \pi/6$.

Angle (X)/radians	Sine X	Cosine X
0	0	1
$\pi/6$	.5	.86603
$\pi/3$	.86603	.5
$\pi/2$	1	0
$2\pi/3$	.86603	-.5
$5\pi/6$	.5	-.86603
π	0	-1

In Question 1, $\sqrt{3}/2$ was used instead of .86603.

3

(a) *Answers in degrees*

- (ii) $\sin 183^\circ = -.052$, $\sin 177^\circ = .052$
 $-\sin(180^\circ + x) = \sin(180^\circ - x)$
- (iii) $\cos 10^\circ = .985$, $\cos -10^\circ = .985$
 $\cos x = \cos(-x)$
- (iv) $\cos 9^\circ = .998$, $\cos 351^\circ = .998$
 $\cos x = \cos(360^\circ - x)$
- (v) $\sin 3^\circ = .052$, $\cos 87^\circ = .052$
 $\sin x = \cos(90^\circ - x)$
- (vi) $\tan 5^\circ = .087$, $\tan 185^\circ = .087$
 $\tan x = \tan(180^\circ + x)$
- (vii) $(\sin 37^\circ) \div (\cos 37^\circ) = \tan 37^\circ = .754$
 $(\sin x) \div (\cos x) = \tan x$
- (viii) $2(\sin 37^\circ)(\cos 37^\circ) = .961$, $\sin 74^\circ = .961$
 $2 \cos x \sin x = \sin 2x$
- (ix) $(\cos 19^\circ)^2 - (\sin 19^\circ)^2 = .788$
 $\cos 38^\circ = .788$
 $(\cos x)^2 - (\sin x)^2 = \cos 2x$
- (x) $(\cos 19^\circ)^2 + (\sin 19^\circ)^2 = 1$
 $(\cos x)^2 + (\sin x)^2 = 1$

(b) *Answers in radians*

- (ii) $-\sin(\pi + x) = \sin(\pi - x)$
- (iii) $\cos x = \cos(-x)$
- (iv) $\cos x = \cos(2\pi - x)$
- (v) $\sin x = \cos(\pi/2 - x)$
- (vi) $\tan x = \tan(\pi + x)$
- (vii) $\sin x / \cos x = \tan x$
- (viii) $2 \sin x \cos x = \sin 2x$
- (ix) $(\cos x)^2 - (\sin x)^2 = \cos 2x$
- (x) $(\cos x)^2 + (\sin x)^2 = 1$

4

- (a) Both graphs have the same amplitude, i.e. height. However, $y = \cos 2x$ has twice as many crests as $y = \cos x$, i.e. it has half the period (or wavelength) of $\cos x$.
- (b) Both graphs have the same period, but the amplitude of $y = \sin 2x$ is twice the amplitude of $y = \sin x \cos x$.
- (c) The graphs of Y1 and Y2 are identical, because $\sin x / \cos x = \tan x$.
- (d) Y1 is the mirror image of Y2 (and, of course, vice versa).
- (e) All values on both graphs are positive. This is because any negative values, when squared, become positive.
 The graphs have the same shape but are moved laterally $\pi/2$ with respect to one another.
- (f) The graph is a straight line which has the value $y = 1$ for all values of x . This is because $(\sin x)^2 + (\cos x)^2 = 1$.

5

- (a) Since 1/18 of the previous string length is marked off at each stage, the effective length of the string is reduced to 17/18 of its previous value, giving

$$\begin{aligned} \text{Rule of 18 string-length ratio} \\ = 17/18 = .9444 \text{ (to 4 d.p.)} \end{aligned}$$

However,

$$\begin{aligned} \text{theoretical string-length ratio} \\ = \sqrt[12]{(1/2)} = 0.9439 \text{ (to 4 d.p.)} \end{aligned}$$

The values agree to 3 decimal places.

(b) A suitable program is

```
PROGRAM:FRETS
:650 → L : 650 → M
:For (C, 1, 21)
:L → L1(C)
:M → L2(C)
:L * R → L
:M * E → M
:End
```

Notice how it is similar in structure to the program called SCALE.

By using this program, the following distances are obtained:

Theoretical distance /mm	Rule of 18 distance /mm
650	650
614	614
579	580
547	548
516	517
487	488
460	461
434	436
409	411
386	389
365	367
344	347
325	327
307	309
290	292
273	276
258	260
243	246
230	232
217	219
205	207

(c) Since the Rule of 18 string-length ratio is slightly larger than the theoretical ratio, it gives rise to slightly longer string lengths and, hence, fret distances. This effect becomes more marked at every stage—but even when the last fret is reached, the discrepancy is only about 2 mm (this corresponds to an interval of about one sixth of a semitone).

6

(a) Multiplying both sides by $2L$ gives

$$2Lf = \sqrt{\frac{T}{M}}.$$

Squaring both sides gives

$$4L^2 f^2 = \frac{T}{M}.$$

Then multiply both sides by M , and thus

$$T = 4L^2 M f^2.$$

(b) (i) The mass of the string is 0.0038 kg, and its length is 0.64 m. The mass per unit length is thus $(0.0038/0.64)$ kg/m, i.e. 0.0059375 kg/m.

(ii) Evaluating the expression for T in part (a), with $L = 0.64$, $M = 0.0059375$ and $f = 256$, gives the required tension, to the nearest newton, as 638 newtons.

A 1 kg weight, hanging on a string, produces a tension of nearly 10 newtons. So the piano string is stretched as tightly as if it were carrying a weight of over 60 kg. Since a piano has a total of about 230 strings, each of which will be under similar tension, the total force on the frame of the piano will be comparable to a weight of between 10 and 20 tonnes—to withstand these enormous forces, the frame is constructed of heavy steel, with considerable bracing.

7

(a) The graph may look something like this, depending on the window settings chosen.

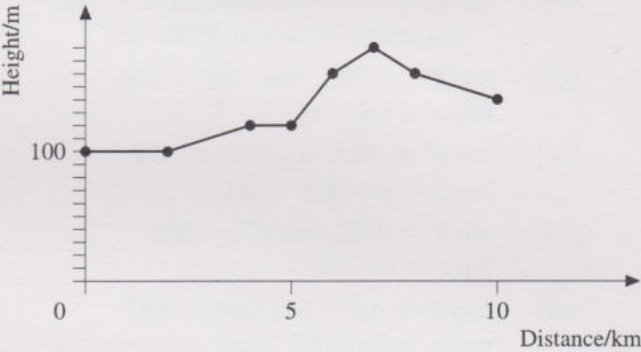


Figure 14

The line graph assumes that the gradients between pairs of consecutive intermediate positions are constants.

(b) $T = 60t$.

Now, from Naismith's rule,

$$60t = \frac{60d}{5} + \frac{60h}{600},$$

so

$$T = 12d + \frac{h}{10}.$$

(c) to (e)

(1) Point on journey	(2) Distance from start/km	(3) Height above sea level/m	(4) Distance from previous point/km
A	0	100	0
B	2	100	2
C	4	120	2
D	5	120	1
E	6	160	1
F	7	180	1
G	8	160	1
H	10	140	2

(5) Height above previous point/m	(6) Time taken from previous point/min	(7) Mean time taken by Ron/min
0	0	0
0	24	12
20	26	16
0	12	6
40	16	14
20	14	10
-20	12	2
-20	24	8

(f) Where $h = 0$ and $h' = 0$, Ron's rule becomes

$$T = Pd,$$

so

$$P = T/d.$$

For the part of the journey from A to B (for which $h = 0$ and $h' = 0$), $d = 2$ and $T = 12$, hence $P = 6$.

The same result is obtained using data for part of the journey from C to D.

(g) Where $h' = 0$, Ron's rule becomes

$$T = Pd + \frac{h}{Q},$$

so, as $P = 6$,

$$Q = h/(T - 6d).$$

For the part of the journey from B to C (for which h increases but $h' = 0$), $h = 20$, $T = 16$ and $d = 2$, so $Q = 5$.

The same result is obtained using data for other parts of the journey where h increases but $h' = 0$.

(h) From Ron's rule, if $h = 0$, then

$$T = Pd - \frac{h'}{R}.$$

But $P = 6$, so

$$R = -h'/(T - 6d).$$

For the part of the journey from F to G (for which $h = 0$), $h' = 20$, $T = 2$ and $d = 1$, so $R = 5$.

Then Ron's rule is

$$T = 6d + \frac{h}{5} - \frac{h'}{5}.$$

(i) Ron's rule:

$$T = 6d + (h - h')/5$$

becomes

$$T = 6d + \frac{w}{5}.$$

(j) For the journey from H back to A, $d = 10$ and $w = -40$, so substituting into Ron's rule from part (i) gives

$$\begin{aligned} T &= 6 \times 10 - 40/5 \\ &= 60 - 8 \\ &= 52. \end{aligned}$$

Therefore, the time calculated for the journey is 52 minutes.

However, going from E to D the gradient is very steep, and Ron's rule predicts a time of -2 minutes. In practice, a cyclist would brake at such points. Certainly the rule would not apply to such sharp inclines.

8

(a) [5, 12, 13] [9, 12, 15].

(b) When $n = 5$, the triple is [5, 12, 13].When $n = 9$, the triple is [9, 40, 41].When $n = 17$, the triple is [17, 144, 145].When $n = 21$, the triple is [21, 220, 221].

$$\begin{aligned} \text{(c) (i) } A^2 + B^2 &= n^2 + ((n^2 - 1)/2)^2 \\ &= n^2 + (n^2 - 1)^2/4 \\ &= n^2 + (n^4 - 2n^2 + 1)/4 \\ &= n^2 + n^4/4 - n^2/2 + 1/4 \\ &= n^4/4 + n^2/2 + 1/4. \end{aligned}$$

$$\begin{aligned} \text{(ii) } C^2 &= ((n^2 + 1)/2)^2 \\ &= (n^2 + 1)^2/4 \\ &= (n^4 + 2n^2 + 1)/4 \\ &= n^4/4 + n^2/2 + 1/4. \end{aligned}$$

(iii) The expression in (i) is the same as the expression in (ii). So $A^2 + B^2 = C^2$ when $A = n$, $B = (n^2 - 1)/2$ and $C = (n^2 + 1)/2$, so $[n, (n^2 - 1)/2, (n^2 + 1)/2]$ is a Pythagorean triple.

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